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ENTROPY GENERATION IN COMBINED HEAT AND MASS TRANSFER EFFECT ON MHD FREE CONVECTION FLOW PAST AN OSCILLATING PLATE

OKEDOYE A. M

Department of Mathematics and Computer Science, Federal University of Petroleum Resources,
P. M. B. 1221 Effurun Delta

ABSTRACT

Analytical calculation of entropy generation due to unsteady magnetohydrodynamic heat and mass transfer in MHD flow past an infinite vertical oscillating plate was considered, taking account of the presence of free convection and mass transfer. The fluid and the plates are in a state of solid body oscillation with constant angular velocity about the z-axis normal to the plates. The energy and chemical species equations are solved in closed form by using separation of variable technique and then perturbation expansion for the momentum equation. The influences of various flow conditions were investigated, reported and discussed.

KEYWORDS: Free Convection, Magnetohydrodynamic Flows, Mass Transfer, Oscillating Plate, Permeable Surface, Porous Medium, Viscosity

Classification: 76W05

INTRODUCTION

The study of Magnetohydrodynamic flow and heat transfer has received considerable attention in recent years due to its wide variety of applications in engineering and technology such as MHD generators, plasma studies, nuclear reactors and geothermal energy extractions. Free convection flows are of great interest in a number of industrial applications such as fibre and granular insulation, geothermal systems etc. Buoyancy is also of importance in an environment where differences between land and air temperatures can give rise to complicated flow patterns. In industries and nature, many transport processes exist in which heat and mass transfer takes place simultaneously as a result of combined buoyancy effect of thermal diffusion and diffusion of chemical species. Unsteady oscillatory free convective flows play an important role in chemical engineering, turbo machinery and aerospace technology such flows arise due to either unsteady motion of a boundary or boundary temperature. Besides, unsteadiness may also be due to oscillatory free stream velocity and temperature.

Convective heat transfer through porous media has been a subject of great interest for the last three decades. Recently, Magyari *et al* (2004) have discussed analytical solutions for unsteady free convection in porous media. The magnetic current in porous media considered by Raptis*et al* (1983) and Geindreau *et al* (2002).

The contemporary trend in the field of heat transfer and thermal designs is the second Law (of Thermodynamics) analysis and its design-related concept of entropy generation minimization. Entropy generation is associated with thermodynamic irreversibility, which is common in all types of heat transfer processes. Different sources of irreversibility are responsible for heat transfer's generation of entropy like heat transfer across finite temperature gradient, viscous effects, characteristics of convective heat transfer, etc. Thus entropy generation depends functionally on the local values of velocity and temperature in the domain of interest. Energy conversion processes are accompanied by an irreversible increase in entropy, which leads to a decrease in available energy.

Nag and Kumar (1989) studied second Law optimization for convective heat transfer through a duct with constant heat flux. In their study, they plotted the variation of entropy generation versus the temperature difference of the bulk flow and the surface using a duty parameter. Shuja and Yilbas(2001a)analyzed the entropy generation in an impinging jet and Shuja et al (2001b, 2002, 2003) consider swirling jet impingement on an adiabatic wall for various flow conditions. The dissipation of energy takes the form of a sum of products of conjugate forces and fluxes associated to the problem under consideration; this was presented by the text of the De Groot (1966). The fluxes are expressed as linear functions of all forces, as constitutive equations, subjected to the reciprocal relations of Onsager. These lead to coupled field equations for the temperature and species concentrations in a given fluid mixture. Interferences between heat and mass transport, at the level of constitutive equations, and the linear theory of non-equilibrium thermodynamics had been formulated as a constitutive theory capable of fully expressing the dependence of all fluxes as a function of all thermodynamic forces. Entropy generation in MagnetoHydroDynamic (MHD) flow of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction was studied and reported by Okedoye et al (2007). Many researchers have worked on entropy generation and Okedoye et al. (2007) has a good review of some of this work.

Despite the effort of the previous researchers, there is emerging needs for determination of the entropy generation due to unsteady heat and mass transfer effect on MHD free convection flow. This paper is presented to provide insight toentropy generation due to unsteady heat and mass transfer effect on MHD free convection flow past an oscillating plate in the presence of heat generation/absorption and chemical reaction.

Formulation of the Problem

We consider unsteady, free convection flow of anincompressible and electrically conducting viscous fluid along an infinite nonconductingvertical flat plate through a porous medium. The *x*axis is takenalong the plate in the vertically upward direction and *y*axis is taken normal tothe plate. A magnetic field of uniform strength B_0 is applied in the direction offlow and the induced magnetic field is neglected. Initially, the plate and the fluidare at same temperature $T_w$in a stationary condition with concentration level C_w at all points. At time *t*> 0 the plate starts oscillating in its own plane with avelocity $U_0 \cos\omega t$. Its temperature is raised to $e^{i\omega t}$ and the concentration level atthe plate is raised to $e^{i\omega t}$. Using the Boussinesq approximation, the governingequations for the flow are given by:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u-\frac{v}{A}u+\frac{g\beta_\tau}{\rho}\left(T-T_\infty\right)+\frac{g\beta_c}{\rho}\left(C-C_\infty\right) \tag{1}$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho c_p}\left(T-T_\infty\right) \tag{2}$$

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial y^2}-\gamma\left(C-C_\infty\right) \tag{3}$$

The initial and boundary conditions are equally given by:

$$\left.\begin{array}{c} u=0,\ T=T_w,\ C=C_w\ \ for\ all\ \ y,\ t\le 0 \\ u=U_0\cos\omega t,\ T=T_\infty+A_1e^{i\omega t},\ C=C_\infty+A_2e^{i\omega t},\ y=0,\ t>0 \\ u=0,\ T=T_\infty,\ C=C_\infty\ \ as\ \ y\to\infty,\ t>0 \end{array}\right\} \tag{4}$$

Where $A_1 = T_w - T_\infty, A_2 = C_w - C_\infty$

Let us introduce the non-dimensional variables:

$$\left.\begin{array}{l} u'=\dfrac{u}{U_0},\ t'=\dfrac{tU_0^2}{v},\ y'=\dfrac{yU_0}{v},\ A'=\dfrac{U_0^2}{v^2} \\ \omega'=\dfrac{\omega v}{U_0^2},\ \ \theta=\dfrac{T-T_\infty}{T_w-T_\infty},\ \ \phi=\dfrac{C-C_\infty}{C_w-C_\infty} \end{array}\right\} \tag{5}$$

Where all the physical variables have their usual meanings

With the help of (5), on dropping primes ($'$) the governing equations (1) – (3) with the boundary conditionsreduce to:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+Gr\tau\theta+Grc\phi-\left(M+\frac{1}{K}\right)u \tag{6}$$

$$\Pr\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial y^2}+\Pr\delta\theta \tag{7}$$

$$Sc\frac{\partial\phi}{\partial t}=\frac{\partial^2\phi}{\partial y^2}-\alpha Sc\phi \tag{8}$$

$$\left.\begin{array}{l} u=0,\ \ \theta=1,\ \ \phi=1,\ for\,all\ \ y,\ \ t\le 0 \\ u=\cos\omega t,\ \theta=e^{i\omega t},\ \ \phi=e^{i\omega t},\ y=0,\ t>0 \\ u\to 0,\ \ \theta\to 0,\ \ \phi\to 0,\ as\,y\to\infty,\ \ t>0 \end{array}\right\} \tag{9}$$

Where the flow parameters are as defined below:

$$\left.\begin{array}{l} Gr\tau=\dfrac{g\beta_\tau(T_w-T_\infty)v}{U_0^3},\ \ Grc=\dfrac{g\beta_c(C_w-C_\infty)v}{U_0^3},\ \ M=\dfrac{\sigma B_0^2 v}{\rho U_0^2} \\ K=\dfrac{U_0^2 A}{v^2},\ \ \Pr=\dfrac{\mu c_p}{k},\ \ \delta=\dfrac{Qv}{U_0^2\rho c_p},\ \ Sc=\dfrac{v}{D},\ \ \alpha=\dfrac{\gamma v}{U_0^2} \end{array}\right\} \tag{10}$$

METHOD OF SOLUTION

Equations (7) and (8) are solvedin closed form, and given as below

$$\theta(y,t) = e^{i\omega t - \sqrt{(i\omega-\delta)Pr}\,y} \tag{11}$$

$$\emptyset(y,t) = e^{i\omega t - \sqrt{(i\omega+\alpha)Pr}\,y} \tag{12}$$

Using (11) and (12) above, equation (6) becomes

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right)u + Grte^{i\omega t - \sqrt{(i\omega-\delta)Pr}\,y} + Grce^{i\omega t - \sqrt{(i\omega+\alpha)Pr}\,y} \tag{13}$$

To solve (13), we seek a perturbation series expansion of the form

$$u(y,t) = u_0(y) + e^{i\omega t}u_1(y) + e^{2i\omega t}u_2(y) + \cdots$$

This is justified since $t$does not appear explicitly in the equation, thus we write:

$$u(y,t) = u_0(y) + e^{i\omega t}u_1(y) + o(e^{2i\omega t}) + ... \tag{14}$$

The equation (13) becomes:

$$\begin{cases} \dfrac{d^2u_0}{dy^2} - \left(M + \dfrac{1}{K}\right)u_0 = 0 \\ u_0(0) = \cos\omega t\,, u_0(y) \to 0 \text{ as } y \to \infty \end{cases} \tag{15}$$

$$\begin{cases} \dfrac{d^2u_1}{dy^2} - \left(M + \dfrac{1}{K} + i\omega\right)u_1 = -Grte^{-\sqrt{(i\omega-\delta)Pr}\,y} + Grce^{-\sqrt{(i\omega+\alpha)Pr}\,y} \\ u_1(0) = 0, u_1(y) \to 0 \text{ as } y \to \infty \end{cases} \tag{16}$$

The solutions to the equations (15), and (16) respectively are:

$$\left.\begin{aligned} u_0(y) &= \cos\omega t\, e^{\sqrt{\left(M+\frac{1}{K}\right)}\,y} \\ u_1(y) &= a_1e^{-ny} + a_3e^{-\sqrt{(i\omega-\delta)Pr}\,y} + a_4e^{-\sqrt{(i\omega+\alpha)Pr}\,y} \end{aligned}\right\} \tag{17}$$

where

$$n = \sqrt{M + \frac{1}{K} + i\omega}, a_3 = \frac{Grt}{n^2 - (i\omega-\delta)Pr}, a_4 = \frac{Grc}{n^2 - (i\omega+\alpha)Sc}, a_1 = -a_3 - a_4$$

Therefore by (14)

$$u(y,t) = \cos\omega t\, e^{\sqrt{\left(M+\frac{1}{K}\right)}\,y} + e^{i\omega t}[a_1e^{-ny} + a_3e^{-\sqrt{(i\omega-\delta)Pr}\,y} + a_4e^{-\sqrt{(i\omega+\alpha)Pr}\,y}]$$

ENTROPY GENERATION RATE

For an incompressible Newtonian fluid, the local entropy generation rate is given by Curtis and Hirschfelder

$$\Gamma = \frac{\mu}{T}\left(\frac{\partial u_i}{\partial x_j}\right)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{1}{T}\sum_{\alpha} J\alpha_i\left(\frac{\partial u_\alpha}{\partial x_i}\right) - \frac{q}{T^2}\left(\frac{\partial T}{\partial x_i}\right)$$
$$-\frac{1}{T}\sum_{\alpha} S_\alpha J\alpha_i\left(\frac{\partial u_\alpha}{\partial x_i}\right) - \frac{1}{T}\sum_{\alpha} K_\alpha \mu_\alpha$$

On the right hand side of the above equation, the first term is due to fluid friction, the second is due to mass diffusion and the third term is due to heat conduction. The fourth term is due to heat transfer induced by mass diffusion and the fifth is due to chemical reactions.

In convective heat and mass transfer and MHD flow, irreversibility arises due to the heat transfer, the viscous effects and the mass transfer. The entropy generation rate is expressed as the sum of contributions due to viscous, thermal and diffusive effects, and thus it depends functionally on the local values of temperature, velocity and concentration in the domain of interest.

Dimensionless terms denoted $\lambda_i\,(1 \le i \le 3)$, and called irreversibilities distribution ratios, are given by:

$$\lambda_1 = \frac{\mu T_0}{k}\left(\frac{a}{L(\Delta T)}\right)^2, \quad \lambda_2 = \frac{RDT_0}{kc_0}\left(\frac{\Delta c}{\Delta T}\right)^2, \quad \lambda_3 = \frac{RD}{k}\left(\frac{\Delta c}{\Delta T}\right)$$

where C_0 and T_0 are respectively the reference concentration and temperature, which are in our case, the bulk concentration and the bulk temperature.

We can now define the followings:

$$\Gamma_{n,h} = \left(\frac{\partial\theta}{\partial y}\right)^2, \quad \Gamma_{n,f} = \lambda_1\left(\frac{\partial u}{\partial y}\right)^2, \quad \Gamma_{n,d}^{c} = \lambda_2\left(\frac{\partial c}{\partial y}\right)^2, \text{ and } \Gamma_{n,d}^{c,T} = \lambda_3\left(\frac{\partial\theta}{\partial y}\right)\left(\frac{\partial c}{\partial y}\right),$$

where $\Gamma_{n,h}$ and $\Gamma_{n,f}$ are thermal and viscous irreversibility respectively, while $\Gamma_{n,d}^{c} + \Gamma_{n,d}^{c,T}$ is the diffusive irreversibility

Thus we defined the dimensionless entropy Generation rate as:

$$\Gamma_n = \left(\frac{\partial\theta}{\partial y}\right)^2 + \lambda_1\left(\frac{\partial u}{\partial y}\right)^2 + \lambda_2\left(\frac{\partial c}{\partial y}\right)^2 + \lambda_3\left(\frac{\partial\theta}{\partial y}\right)\left(\frac{\partial c}{\partial y}\right) \tag{19}$$

Using the (19) above, on substituting equations (11), (12)and (18) into (19) for irreversibilities, we have:

$$\begin{aligned}\Gamma n = (i\omega-\delta)\,\mathrm{Pr}\left(e^{i\omega t-\sqrt{(i\omega-\delta)Pr}\,y}\right)^2 \\ + \lambda_1\Bigg(-\cos\omega t\,\sqrt{\left(M+\frac{1}{K}\right)}\,e^{\sqrt{\left(M+\frac{1}{K}\right)}\,y} \\ + e^{i\omega t}\left[-na_1e^{-ny} - a_3\sqrt{(i\omega-\delta)Pr}\,e^{-\sqrt{(i\omega-\delta)Pr}\,y} - a_4\sqrt{(i\omega+\alpha)Sc}\,e^{-\sqrt{(i\omega+\alpha)Sc}\,y}\right]\Bigg)^2 \\ + \lambda_2(i\omega+\alpha)Sc\left(e^{i\omega t-\sqrt{(i\omega+\alpha)Pr}\,y}\right)^2 + \lambda_3\sqrt{(i\omega-\delta)Pr} * \sqrt{(i\omega+\alpha)Sc}\,e^{2i\omega t-\left(\sqrt{(i\omega-\delta)Pr}\pm\sqrt{(i\omega+\alpha)Pr}\right)y}\end{aligned}$$

DISCUSSIONS

In order to point out the effects of various parameters on flowcharacteristic, the following discussion is set out.

The values of the Prandtlnumber is chosen Pr = 0.71 (plasma). The values of the Schmidtnumber is chosen to represent the presence of species by water vapour (0.62). All other parameters are primarily chosen as follows: $Gr\tau = 10,\ Grc = 5,\ \alpha = 5,\ B = -2,\ M = 0.5,\ K = 1.5,\ \omega t = 2\pi,\ t = 0.25,$ unless otherwise stated.

We displayed the effect of each parameter on the entropy generation in Fig. 1 – 7. It should be noted that $Grt > 0$ implies cooling of the plate while $Grt < 0$ indicate heating of the plate.In our discussion we assume $Grt > 0$.

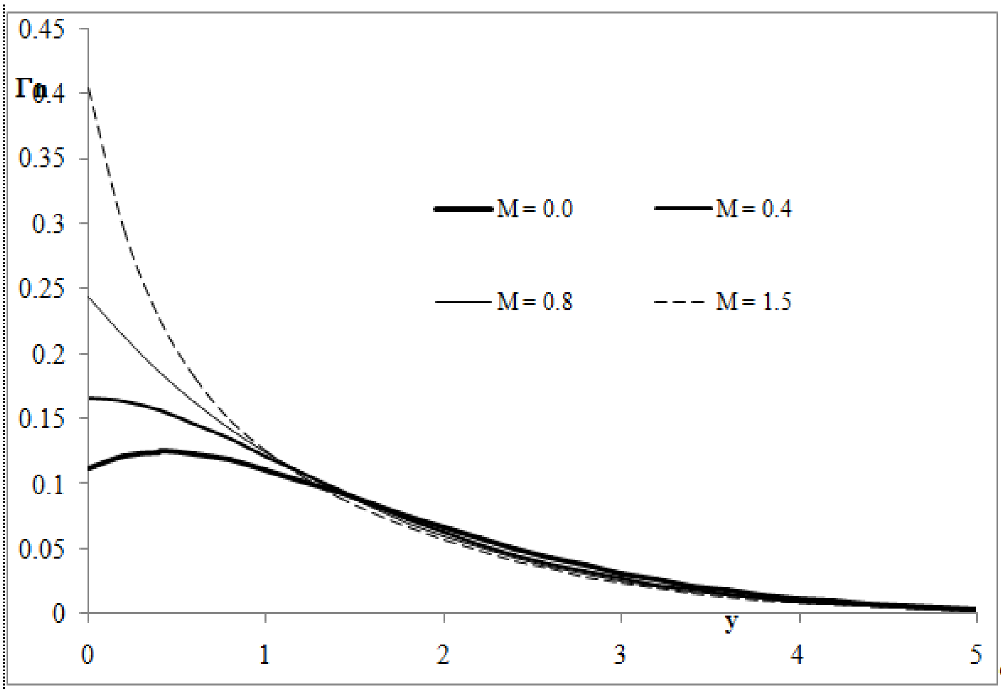

Figure 1: Entropy Generation for Various Value of M

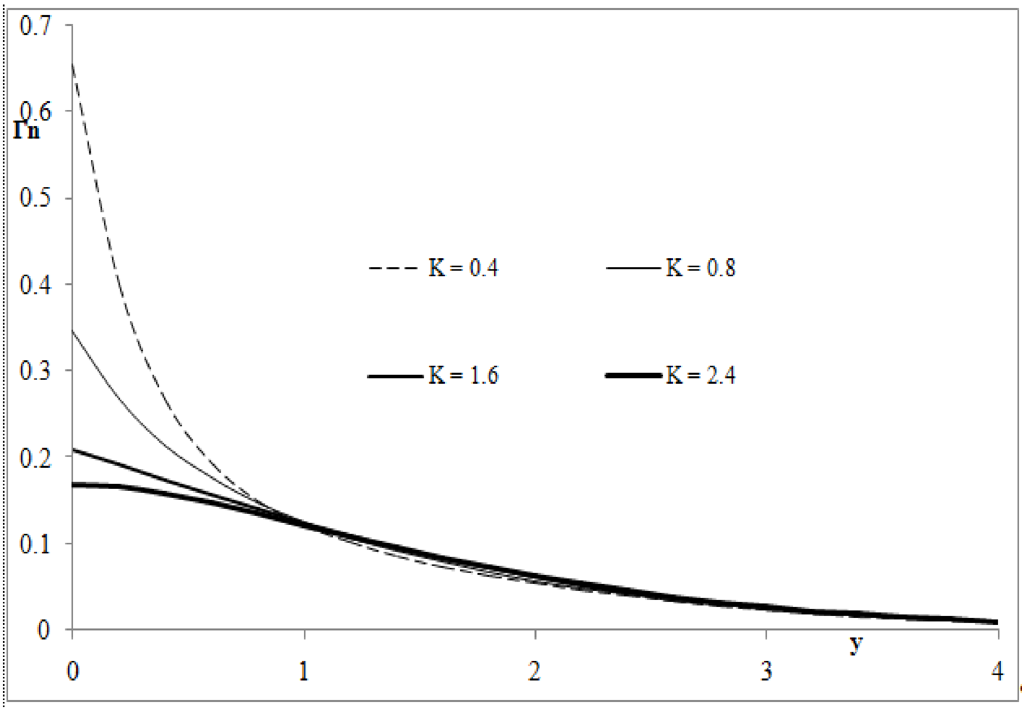

Figure 2: Entropy Generation for Various Values of K

In Figure. 1, we show the effect of magnetic parameter on the entropy generation. It could be seen that entropy increase with an increase in the Hartmann's number. This is because the entropy generation depends on the rate of disorderliness which in turn depends on the magnitude of the velocity. The induced opposing force, the Lorentz force brings about a change in the orientation of the flow which transforms to increase in entropy. It could also be seen that the effect of Hartmann number on entropy is more pronounced for $0 \leq y \leq 1$ as shown in fig 1. When $y > 1$ there is only little change in the numerical value of Γn with respect to changes in M. And far away from the plate Γn tends to zero with little changes due to M. In particular, at $y = 2$for 100% increase in Mwe observe that entropy only increases by 5%.

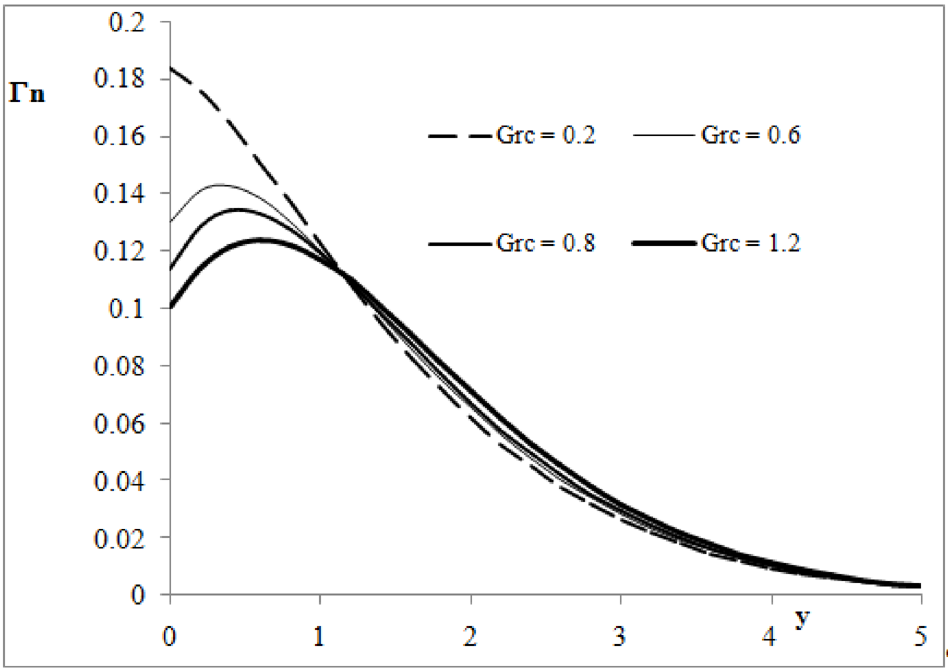

Figure 3: Entropy Generation for Various Values of Grc

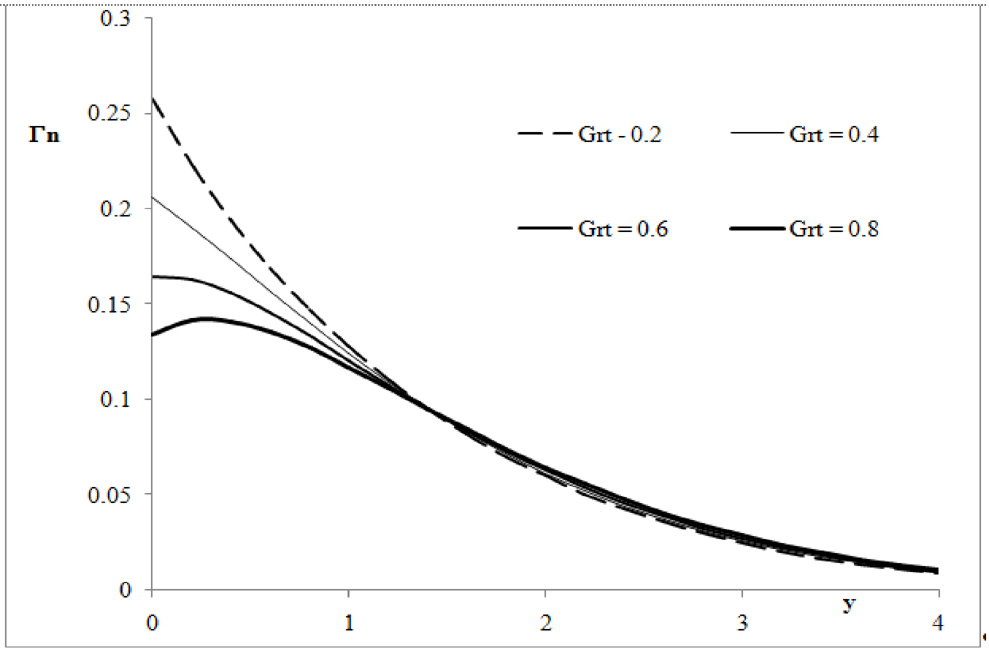

Figure 4: Entropy Generation for Various Values of Grt

Figure 2 depict the effect of permeability parameter on the entropy generation. We observed that entropy generation increases with decrease in permeability factor. This is due to the fact that increase in the value of *K* has the tendency to reduce the thermal and mass buoyancy effect. This gives rise to decrease in the induced flow.

Figure 3 displayed the effect of mass Grashof number on the entropy generation during the flow process. It is observed that higher mass buoyancy, results in an decrease in the entropy. It is due to the fact increase in the values of mass Grashof number and modified Grashof number has the tendency to increase the mass buoyancy effect. This gives rise to andecrease in the induced flow and hence the entropy.

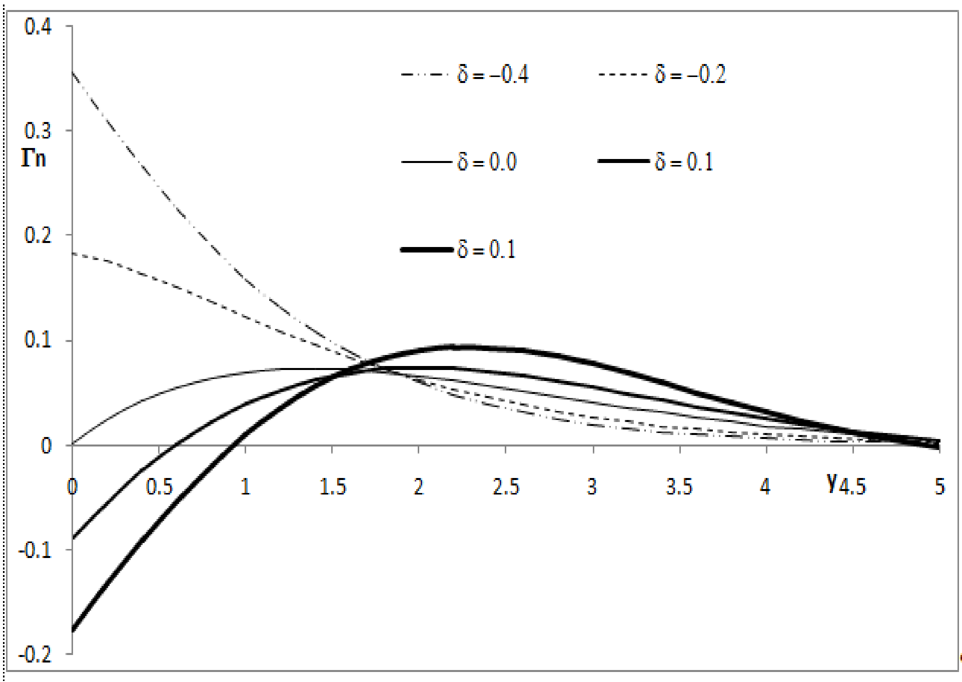

Figure 5: Entropy Generation for Various Values of δ

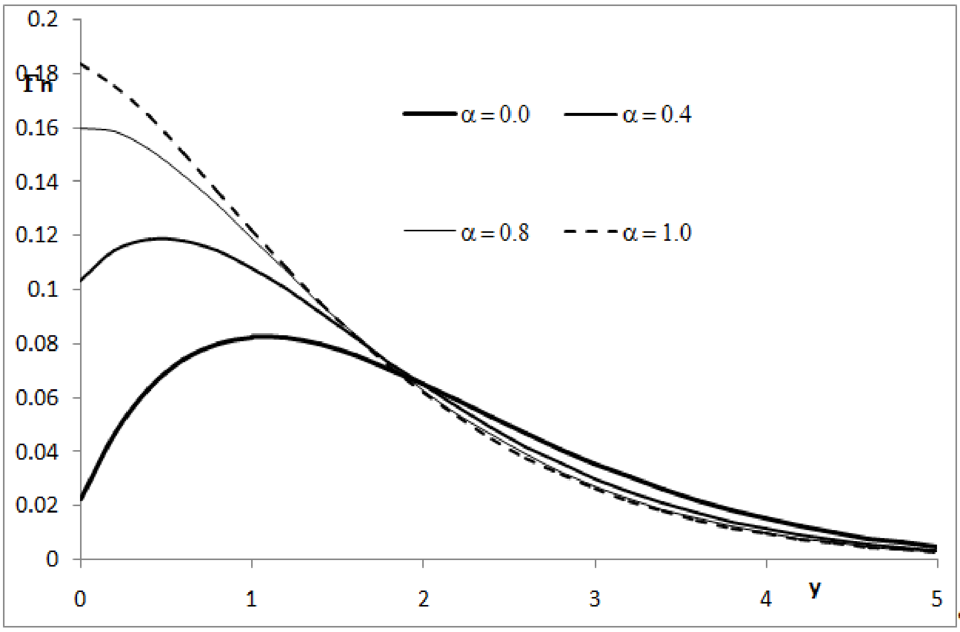

Figure 6: Entropy Generation for Various Values of α

In Figure 4 we displayed the effect of thermal Grashof number on the entropy generation. It is observed that increase in thermal Grashof number results in decrease in the entropy generation. For higher thermal Grashof number, there is practically little or no convection and the entropy generation due to fluid friction is zero, consequently the total entropy generation is reduced to the entropy generation due to heat transfer. At lower Grashof number heat transfer due to convection begins to play a significant role increasing the flow velocity and in turn the entropy generation due to the viscous effects. Also the isotherms are deformed, increasing the temperature gradient and consequently the entropy generation due to heat transfer.

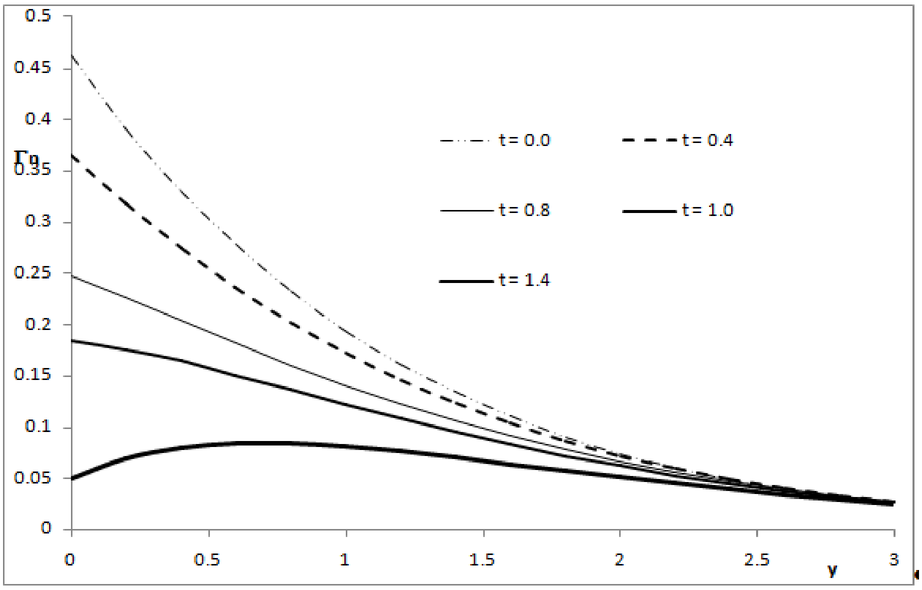

Figure 7: Entropy Generation for Various Values of t

Figure 5 shows the effect of heat generation or absorption on the entropy generation. It could be seen that increase in heat generation $\delta < 0$results in increase in entropy generation. While increase in heat absorption $\delta > 0$ brings about reduction in $\Gamma n.$ More so, heat generation increases the thermal buoyancy and hence increase $\Gamma n.$ We observe that the phenomenon is reversed when $y \in (1.8\ 4.6)$ In Figure 6 we display the effect of chemical reaction parameter on the entropy generation. It is seen that generative chemical reaction increases the entropy generation close to the plate, while far away from the plate it result in decrease in the entropy.

This is because the presence of a porous medium increases the resistance to flow resulting in decrease in the flow entropy. This behaviour is depicted by the decrease in the entropy as the flow position increases above one (2) unit ($y > 2$ Figure (6)) the entropy is lower in the flow field. The effect of unsteadiness in the flow field is shown in Figure (7). The entropy generationdecreases with time faster when $y \in (0, 2)$ after which it decay asymptotically to zero

CONCLUSIONS

In this paper, entropy generation of unsteady heat and mass transfer effect on MHD free convection flow past an oscillating plate in the presence of heat generation/absorption and chemical reactionispresented. Results are presented graphically to illustrate the variation in entropy as a result of variation of the flow control parameters. In this study, the following conclusions are set out:

- For of cooling of the plate (Grt> 0), the entropyincreases with anincrease in magnetic parameter and heat generation, thermal and mass buoyancy. On the other hand, it decreases with an increase in thevalue of reaction factor, heat absorptionpermeabilityparameter and time.

- More so, the effect of flow condition is significant close to the wall. But far away from the wall, it became less significant varying the value of flow condition. In particular when $y > 1$ for M, K, Grc and Grt, and when $y > 2$ for δ and α we observe a little changes in entropy with flow conditions.

Nomenclature

c non- dimensional concentration

D is mass diffusivity

T fluid temperature

u fluid axial velocity

Sc Schmidt number

C_f skin – friction coefficient

t time

$i = \sqrt{-1}$ complex identity

v fluid transverse velocity

c_p specific heat at constant pressure

y transverse or horizontal coordinate

C_w concentration at the wall

T_w temperature at the wall

Greek Symbols

θ non - dimensional fluid temperature

δ heat generation/absorption coefficient

ϕnon – dimensional chemical species

α reaction parameter

ω angular velocity

β_τ coefficient of thermal expansion

β_c coefficient of concentration expansion

Dimensionless Group

Grt dimensionless thermal Grashof number

Grc dimensionless mass Grashof number

M Hartmann number

Pr Prandtl number

Subscripts

w condition on the wall

∞ ambient condition

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